

# BLACK HOLES AND STRING THEORY

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## ABSTRACT

This is a short summary of my lectures given at the Fourth Mexican School on Gravitation and Mathematical Physics. These lectures gave a brief introduction to black holes in string theory, in which I primarily focussed on describing some of the recent calculations of black hole entropy using the statistical mechanics of D-brane states. The following overview will also provide the interested students with an introduction to the relevant literature.

## 1. Prologue

String theory is a very broad and extremely rich area of study in theoretical physics pursued by particle physicists, mathematicians, and relativists as well. Within this community of string theorists, there has long been a fascination with black holes, and studies of the latter have taken many different points of view, including:

1. Black holes with string theory corrections [1]
2. Black holes with quantum hair [2]
3. Two-dimensional black holes as WZW models [3]
4. Black holes in solvable models of two-dimensional gravity [4]
5. Black holes and entanglement entropy [5]
6. Black holes as low energy supergravity solutions [6]
7. Black holes as exact sigma model backgrounds [7]
8. Black holes as strings [8]
9. Black holes as D-branes [9, 10, 11]
10. Black holes in Matrix theory [12]
11. Black holes in the AdS/CFT correspondence [13]
12. Black holes as superconformal quantum mechanics [14]
13. Black holes in brane world scenarios [15]
14. Black holes and enhançon physics [16]

The references cited above are by no means complete. Consulting Paul Ginsparg's e-print archive [17], one finds that in the past ten years, the high energy theory (hep-th) section has accumulated in excess of 1600 papers about black holes. Above, I

have only listed a few reviews or salient articles for each topic to give the reader a bridgehead into the relevant literature. The interested students are encouraged to explore the associated references and citations of these papers with the Spires HEP database [18].

Clearly, I could not hope to tell the full story of black holes and string theory in two hour-long lectures. Instead I only attempted to introduce the students to the ninth item on the list above. That is, I described some of the recent calculations of black hole entropy using techniques involving D-branes. In particular, I focussed on the original calculations of Strominger and Vafa [19]. These were the first calculations of any sort which successfully determined the Bekenstein-Hawking entropy with a statistical mechanical model in terms of some underlying microphysical states. There are already several extensive reviews of the D-brane description of black hole microphysics. In particular, I would recommend those by Peet [10] and by Das and Mathur [11]. I would also highly recommend Juan Maldacena's Ph.D. thesis [9] as a well-written and pedagogical introduction to this topic. With regards to further background references, Clifford Johnson's review [20] of D-brane physics is very good. For a general introduction to modern string theory, the standard reference is now Polchinski's text [21]. Interested students may also wish to look at a similar but longer series of lectures on black holes in string theory, which I presented in Jerusalem in the previous year [22].

## 2. Summary

In the early seventies, Bekenstein [23] made the bold conjecture that black holes carry an intrinsic entropy given by the surface area of the horizon measured in Planck units multiplied by a dimensionless number of order one. In part, this conjecture was motivated by Hawking's area theorem [24] which had shown that, like entropy, the horizon area of a black hole can never decrease in general relativity.

The next crucial insight came from Hawking while investigating quantum fields in a black hole spacetime [25]. He found that external observers detect the emission of thermal radiation from a black hole with a temperature proportional to its surface gravity\*,  $\kappa$ :

$$k_B T = \frac{\hbar \kappa}{2\pi c} \quad . \quad (1)$$

For a Schwarzschild black hole,  $\kappa = c^4/(4GM)$  and so one finds that Hawking's result typically corresponds to an incredibly small temperature:  $T \sim 10^{-7} K$  for a solar mass black hole.

Previously, extensive studies of solutions of Einstein's equations had culminated in the formulation of four laws of black hole mechanics[27]. Hawking's discovery of a black hole temperature was the key to realizing that these previous results were the laws of thermodynamics applied to black holes. For example, there is a correspon-

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\*The surface gravity may be thought of as the redshifted acceleration of a fiducial observer moving just outside the horizon [26].

dence between the first law in each of these frameworks:

$$\frac{c^2 \kappa}{8\pi G} \delta A = c^2 \delta M \quad \longleftrightarrow \quad T \delta S = \delta U . \quad (2)$$

Here,  $Mc^2$  is naturally identified with the black hole's internal energy,  $U$ . Hence given Hawking's relation (1) between the surface gravity and the temperature, the correspondence between these two relations is completed by identifying [25]

$$S = \frac{k_B c^3}{\hbar G} \frac{A}{4} , \quad (3)$$

which gives a precise relation confirming Bekenstein's earlier conjecture.

However, these revelations about the thermal nature of black holes lead to two related puzzles. The above discussion describes black hole entropy within the framework of thermodynamics, where it is associated with the energy in a system which is unavailable to do work, *e.g.*, in eq. (2),  $T \delta S$  indicates the heat loss in some process. For ordinary thermal systems, statistical mechanics provides a complementary interpretation of entropy by taking into account the microscopic degrees of freedom of the system. In this context, entropy has quite a different significance. It is a measure of the lack of detailed information about the microphysical state of a system. However, in the case of black holes, it remained a longstanding problem to find a statistical mechanical derivation of the entropy.

An even more dramatic puzzle is the black hole information loss paradox. Classical general relativity says that whatever falls into a black hole cannot afterwards be observed from the outside. In principle though, we could discover what fell in by entering the black hole ourselves. However if quantum processes cause the black hole to radiate away its energy thermally so that eventually the black hole disappears, then the information about what has fallen in is completely lost. In fact, such a loss of information violates unitary time evolution, one of the basic tenets of quantum mechanics, the theory which lead to the black hole evaporation in the first place. This paradox has profound implications as it was originally suggested to indicate that quantum mechanics and general relativity simply can not be combined in a consistent manner. It was long felt that a resolution of either of these puzzles would yield some insight into the nature of quantum gravity. This is the essential source of the fascination which string theorists and particle physicists have for black holes.

Recently, progress into these questions has been made with new insights from string theory. This progress is a spin-off from the research into string dualities [28] and the realization of the important role of extended objects beyond just strings [29]. In particular, a class of extended objects known as Dirichlet branes or D-branes [20] have proven very valuable from a calculational standpoint. These objects have a simple description in the framework of perturbative or weakly-interacting strings, and yet they exhibit rich dynamics, including a wide variety of complicated bound states.

In the low energy or long wavelength limit, string theory is accurately described by Einstein gravity coupled to various kinds of matter. In my lectures, I focussed on what is known as the Type IIB superstring theory. In this case, one has a ten-dimensional

supergravity theory where the matter fields include two scalars (the dilaton and the axion), two two-form potentials, a four-form potential and various fermions. As we saw in Don Marolf's lectures [30], various kinds of extended objects can carry charges under the form fields. The fundamental strings of the theory act as the electric sources for one of the two-form potentials, known as the NS (Neveu-Schwarz) two-form. The other potential, the RR (Ramond-Ramond) two-form, has electric sources known as D1-branes, and magnetic sources known as D5-branes (*i.e.*, these are D-branes extended in one and five spatial dimensions, respectively). From the full quantum string theory, we know there is an analog of Dirac charge quantization for ordinary electric charges and magnetic monopoles in four dimensions [31], which requires that the RR two-form charges come in discrete units [29]. Hence if a system carries a certain RR charge, one can use the charge to count the total number of constituent D-branes that must have been used in assembling the system.

In the lectures, I focussed on a particular family of black hole solutions in the Type IIB supergravity theory. From a ten-dimensional point of view, these solutions describe black five-branes carrying three distinct types of charge, including both electric and magnetic charge with respect to the RR two-form. Hence we can say that the black brane was formed by bringing together some number of D1-branes and D5-branes,  $N_1$  and  $N_5$ . Furthermore, the details of the solution allow us to infer that all of the D5-branes were arranged in parallel on a common five-dimensional hypersurface and that, similarly, the D1-branes were parallel on a common line in this surface. In order that the resulting black brane has a finite mass, charge and horizon area, we imagine that the directions in the above hypersurface are wrapped on circles to form a five-dimensional torus. The third charge is a momentum along the circle common to the D1-branes and D5-branes. A standard result of KK (Kaluza-Klein) theory is that such an internal momentum must also be quantized [32], and so we use  $N_P$  to denote the number of momentum quanta carried by the solution. From the point of view of the effective five-dimensional theory, the Penrose diagram for these solutions is similar to that of the Reissner-Nordstrom solution in four dimensions [26]. The solution presented is distinguished by the fact that there is a supersymmetric limit in which the horizon area remains finite and the black brane becomes extremal (*i.e.*, the surface gravity and hence the Hawking temperature vanish). Identifying black hole solutions with these properties is nontrivial as can be seen by the fact that if any of the three charges is set to zero, the horizon is replaced by a null singularity in the supersymmetric limit. Evaluating the black hole entropy according to the Bekenstein-Hawking formula (3) yields

$$S = 2\pi\sqrt{N_1 N_5 N_P} . \quad (4)$$

Note that the right hand side is a pure number that does not depend, *e.g.*, on the details of the compactification to five dimensions.

This result (4) for the classical supergravity solution relies on two inputs from the underlying Type IIB string theory. The first was the charge quantization conditions alluded to above, and the second was a formula for Newton's constant in ten dimensions:  $16\pi G = (2\pi)^7 g_s^2 \ell_s^8$ . Here Newton's constant is expressed in terms of two

parameters arising in the perturbative string theory:  $g_s$ , the string coupling constant, a dimensionless parameter which describes the strength with which the fundamental (closed) strings interact, and  $\ell_s$ , the string scale which can be regarded as the typical size of a fundamental string.

As mentioned previously, the D-branes can also be analysed using the techniques of perturbative string theory. From this point of view, one is considering a particular bound state of  $N_1$  D1-branes and  $N_5$  D5-branes. The  $N_P$  units of momentum are carried by fundamental strings connecting the D1-branes and D5-branes. It should be evident that there are of the order of  $N_1 N_5$  different species of strings that can serve in this role, and further that this momentum may be partitioned amongst the various string excitations in many different ways, *i.e.*, an individual string may carry anywhere between 1 and  $N_P$  units of momentum. Therefore the supersymmetric ground state of this bound state has a large degeneracy,  $\mathcal{D}$ . Given this degeneracy, one can assign a statistical mechanical entropy to the system according to  $S = \log \mathcal{D}$ . A precise evaluation of the degeneracy then exactly reproduces the entropy given in eq. (4). Hence this calculation yields a striking agreement between the Hawking-Bekenstein entropy and the statistical entropy of the D-brane microstates.

Now at this point, the attentive student must have been asking: what does a calculation of the degeneracy of a D-brane bound state have to do with a calculation of the Hawking-Bekenstein entropy of a black hole solution? The relation between these calculations is that the D-brane bound state in perturbative string theory and the black hole in supergravity are actually complementary descriptions of the same system, valid in different regimes of the coupling. Despite their simple description in perturbative string theory, D-branes are nonperturbative objects. This can be seen from the tension (energy density) for a single D-brane which is inversely proportional to the string coupling,  $T \sim 1/g_s$ . However, the gravitational footprint may still be small as this involves coupling the D-brane to gravity with Newton's constant. With the result quoted above, one finds, for example, for a collection of D5-branes:  $r_g^2 = GN_5 T \sim N_5 g_s \ell_s^2$ . This radius  $r_g$  can be regarded as the length scale over which the curvatures and other fields are strong. Hence in a regime where  $N_5 g_s \ll 1$ ,  $r_g$  is less than the string scale  $\ell_s$  and so is a distance that we can't expect to resolve effectively in the perturbative string theory. In this regime, the D-branes are effectively described by a perturbative picture where the D-branes are represented by a source in flat empty space which couples weakly to the fundamental strings. On the other hand, in the regime  $N_5 g_s \gg 1$ ,  $r_g$  is much larger than the string scale  $\ell_s$  and a nonlinear supergravity solution providing a background for the propagation of the strings is a better description of the physics. Note that this "strong coupling" regime may still have  $g_s \ll 1$ . Then the fundamental strings couple weakly to each other but interact strongly with the collection of D-branes.

Therefore the perturbative string picture and the black hole solution provide complementary pictures of the same D1-D5 bound state. One point of view is that when  $g_s$  (and hence Newton's constant) is increased above the weak coupling regime, the system undergoes gravitational collapse forming a black hole. The final step in the argument is that the system under consideration is supersymmetric [33]. Supersymmetry plays an essential role here in that it guarantees that the number of ground

states is independent of the strength of the string coupling, *i.e.*, the ground state degeneracy is a kinematic quantity rather than a dynamical quantity. Therefore one can expect that the calculations in both regimes will yield the same result.

The original calculations of the five-dimensional black hole [19] were quickly extended to spinning black holes [34], four-dimensional black holes [35] and also near-extremal black holes [36]. In the latter case, where the temperature is slightly greater than zero, one can develop a D-brane model for the process of Hawking evaporation [37]. This microscopic model [38] even captures the grey body factors which modify the thermal spectrum for the particles radiated to asymptotic infinity [39]. For the near-extremal calculations, one has lost supersymmetry and so one must modify the arguments which relate the results in the weak and strong coupling regimes [40]. Further, the robustness of these D-brane models is related to the AdS/CFT correspondence [13], which comes into play in describing the near horizon region of the five-dimensional black holes.

These results represented a major breakthrough in our understanding of black hole microphysics, as a statistical mechanical interpretation of black hole entropy had eluded theoretical physicists for over 20 years following the discovery of Hawking radiation. While the paradox of information loss in black hole evaporation remains unresolved, D-branes seem to provide a robust model of at least certain evaporating black holes and so the tools to resolve this perplexing paradox seem to be at hand. In any event, the present remarkable calculations already provide further sanction for string theory as the theory to reconcile quantum mechanics and general relativity.

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